Asymptotic Connectivity of Low Duty-Cycled Wireless Sensor Networks

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I. INTRODUCTION

The Army's Future Combat Systems potentially rely heavily on the efficient use of unattended sensors to detect, identify and track targets in order to enhance situation awareness, agility and survivability. Among different types of sensors, the unattended ground sensors (UGS) are typically deployed and left to self-organize and carry out various sensing, monitoring, surveillance and communication tasks. These sensors are operated on battery power, and energy is not always renewable due to cost, environmental and form-size concerns. This imposes a stringent energy constraint on the design of the communication architecture, communication protocols, and the deployment and operation of these sensors. It is thus critical to operate these sensors in a highly energy efficient manner.

It has been observed that low power sensors consume significant amount of energy while idling in addition to that consumed during transmission and reception. Consequently, it has been widely considered a key method of energy conservation to turn off sensors that are not actively involved in sensing or communication. By functioning at a *low duty cycle*, i.e., by reducing the fraction of time that a sensor is active/on, sensors are able to conserve energy, which consequently leads to prolonged lifetime. This is particularly applicable in scenarios where sensors are naturally idle for most of the time (e.g., detection of infrequent events such as fire, fault, etc., and transmission of very short messages).

However, as sensors alternate between sleep and wake modes, its coverage and communication capability are inevitably disrupted. Duty-cycling sensory devices directly leads to loss of sensing coverage, while duty-cycling radio transceivers directly leads to loss of network connectivity. It is therefore crucial to understand the performance degradation as a result of duty-cycling the sensor nodes, and to design good networking mechanisms that work well with low duty-cycled sensor networks.

In this paper we aim at understanding the fundamental relationship between duty-cycling the radio transceivers and the resulting network connectivity. [1] studied connectivity problems in a network with n nodes independently and randomly placed in a unit square according to a uniform distribution. It showed that it is sufficient and necessary for each node to be connected to $\Theta(\log n)$ nearest neighbors to achieve asymptotic connectivity as n approaches infinity. Building on this result, in this study we consider the same network with the difference that each node is awake/active with probability $p(n)$ and is connected to active neighbors within the range of transmission $R(n)$ when it is awake. We show how $p(n)$ and $R(n)$ are related to ensure network connectivity as n approaches infinity.

II. MAIN RESULTS

Let $\mathcal{G}_p(n, R(n))$ denote a network formed in a unit square where n nodes are deployed randomly and independently according to a uniform distribution. Each node is active with probability $p(n)$ and is connected to active

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14. ABSTRACT

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neighbors within the range of $R(n)$. We are interested in the asymptotic connectivity of $\mathcal{G}_p(n, R(n))$, which is defined as the existence of a route from every active node to every other active node in $\mathcal{G}_p(n, R(n))$ when n approaches infinity. In particular, our asymptotic regime implies not only that n approaches infinity but also that $np(n)$ approaches infinity. Our main result is summarized in the following theorem.

Theorem 1 *The network $\mathcal{G}_p(n, R(n))$ is asymptotically connected with probability one if and only if $p(n)R(n)^2 = \Theta(\frac{\log(np(n))}{n})$. Specifically, there exist two constants $0 < k_1 < k_2$ such that:*

$$\lim_{n \to \infty} Pr\{\mathcal{G}_p(n, R(n)) \text{ is disconnected when}$$
$$np(n)R^2(n) = k_1 \log(np(n))\} = 1,$$

and

$$\lim_{n \to \infty} Pr\{\mathcal{G}_p(n, R(n)) \text{ is connected when}$$
$$np(n)R^2(n) = k_2 \log(np(n))\} = 1.$$

The proof of this theorem utilizes results from [1] as well as the following types of networks. (1) $\mathcal{G}^\lambda(n, R(n))$: a network where nodes are deployed according to a Poisson point process with intensity n and each node is connected to all other nodes within fixed transmission range $R(n)$; (2) $\mathcal{F}^\lambda(n, \phi_n)$: a network where nodes are deployed according to a Poisson point process with intensity n and each node is connected to ϕ_n closest neighbors; (3) $\mathcal{F}(n, \phi_n)$: a network where n nodes are deployed independently and uniformly, and each is connected to ϕ_n closest neighbors.

The detailed proof will be provided in the full version of the paper. In this abstract, a sketch of the proof is given as follows. First we show that given $p(n)R^2(n) = \Theta(\frac{\log(np(n))}{n})$, $\mathcal{G}_p(n, R(n))$ is asymptotically connected if $\mathcal{G}^\lambda(np(n), R(n))$ is asymptotically connected. This is proven by using Poisson distribution to approximate Binomial distribution. Next we show that given $p(n)R^2(n) = \Theta(\frac{\log(np(n))}{n})$, $\mathcal{G}^\lambda(np(n), R(n))$ is asymptotically connected if $\mathcal{F}^\lambda(np(n), \phi_n)$

with $\phi_n = \Theta(\log(np(n)))$ is asymptotically connected. The basic idea behind the proof is to show that given $p(n)R^2(n) = \Theta(\frac{\log(np(n))}{n})$ each circle of radius $R(n)$ in $\mathcal{G}^\lambda(np(n), R(n))$ contains $\Theta(\log(np(n)))$ number of neighbors. Finally, we show that $\mathcal{F}^\lambda(np(n), \phi_n)$ with $\phi_n = \Theta(\log(np(n)))$ neighbors is asymptotically connected with probability one. This proof is obtained via a slight modification to the proof of asymptotic connectivity for $\mathcal{F}(n, \phi_n)$ in [1].

III. DISCUSSIONS

In this paper we discuss in detail a number of implications of this result, as well as related issues. We first discuss the main contribution of this paper in addition to the results in [1] and a similar problem considered in [2] (a special case of the grid deployment). In addition, asymptotic connectivity in a sparse network (with fixed density but increasing number of nodes and area) is discussed, and compared to the dense network scenario considered in this paper. We also discuss some generalized definitions of asymptotic connectivity to investigate how the sufficient and necessary conditions are affected. Finally we show some experimental results on the phase transitions of connectivity probability in a low duty-cycled sensor network.

IV. CONCLUSION

In this paper we showed that in a low duty-cycled wireless sensor network, the necessary and sufficient conditions for asymptotic connectivity are $p(n)R^2(n) = \Theta(\frac{\log(np(n))}{n})$. It is evident from this result that in order to ensure asymptotic connectivity, the required number of active/wake neighbors is on the same order as the required number of neighbors in a network without duty-cycling.

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